PURPLE COMET MATH MEET– April 2007
MIDDLE SCHOOL – PROBLEMS

Problem 1

Last Sunday at noon the date on the calendar was 15 (April 15, 2007). What will be the date on the calendar one million minutes after that time?

Problem 2

How many numbers n have the property that both $\frac{n}{2}$ and $2n$ are four digits whole numbers?

Problem 3

Square ABCD has side length 36. Point E is on side AB a distance 12 from B, point F is the midpoint of side BC, and point G is on side CD a distance 12 from C. Find the area of the region that lies inside triangle EFG and outside triangle AFD.

Problem 4

Terry drove along a scenic road using 9 gallons of gasoline. Then Terry went onto the freeway and used 17 gallons of gasoline. Assuming that Terry gets 6.5 miles per gallon better gas mileage on the freeway than on the scenic road, and Terry's average gas mileage for the entire trip was 30 miles per gallon, find the number of miles Terry drove.

Problem 5

The repeating decimal 0.328181818181... can equivalently be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

Problem 6

The product of two positive numbers is equal to 50 times their sum and 75 times their difference. Find their sum.

Problem 7

There is an interval $[a, b]$ that is the solution to the inequality $|3x - 80| \le |2x - 105|$. Find $a + b$.

Problem 8

Penelope plays a game where she adds 25 points to her score each time she wins a game and deducts 13 points from her score each time she loses a game. Starting with a score of zero, Penelope plays m games and has a total score of 2007 points. What is the smallest possible value for m?

Problem 9

Purple College keeps a careful count of its students as they progress each year from the freshman class to the sophomore class to the junior class and, finally, to the senior class. Each year at the college one third of the freshman class drops out of school, 40 students in the sophomore class drop out of school, and one tenth of the junior class drops out of school. Given that the college only admits new freshman students, and that it wants to begin each school year with 3400 students enrolled, how many students does it need to admit into the freshman class each year?

Problem 10

Tom can run to Beth's house in 63 minutes. Beth can run to Tom's house in 84 minutes. At noon Tom starts running from his house toward Beth's house while at the same time Beth starts running from her house toward Tom's house. When they meet, they both run at Beth's speed back to Beth's house. At how many minutes after noon will they arrive at Beth's house?

Problem 11

The alphabet in its natural order `ABCDEFGHIJKLMNOPQRSTUVWXYZ` is T_0. We apply a permutation to T_0 to get T_1 which is `JQOWIPANTZRCVMYEGSHUFDKBLX`. If we apply the same permutation to T_1, we get T_2 which is `ZGYKTEJMUXSODVLIAHNFPWRQCB`. We continually apply this permutation to each T_m to get T_{m+1}. Find the smallest positive integer n so that $T_n = T_0$.

Problem 12

If you alphabetize all of the distinguishable rearrangements of the letters in the word **PURPLE**, find the number n such that the word **PURPLE** is the n-th item in the list.

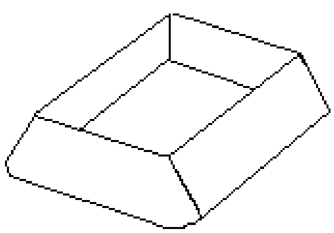
Problem 13

Evaluate the sum

$$1^2 + 2^2 - 3^2 - 4^2 + 5^2 + 6^2 - 7^2 - 8^2 + \cdots - 1000^2 + 1001^2.$$

Problem 14

A rectangular storage bin measures 10 feet by 12 feet, is 3 feet tall, and sits on a flat plane. A pile of dirt is pushed up against the outside of the storage bin so that it slants down from the top of the storage bin to points on the ground 4 feet away from the base of the storage bin as shown. The number of cubic feet of dirt needed to form the pile can be written as $m + n\pi$ where m and n are positive integers. Find $m + n$.



Problem 15

We have some identical paper squares which are black on one side of the sheet and white on the other side. We can join nine squares together to make a 3 by 3 sheet of squares by placing each of the nine squares either white side up or black side up. Two of these 3 by 3 sheets are distinguishable if neither can be made to look like the other by rotating the sheet or by turning it over. How many distinguishable 3 by 3 squares can we form?